

1- INTEGERS

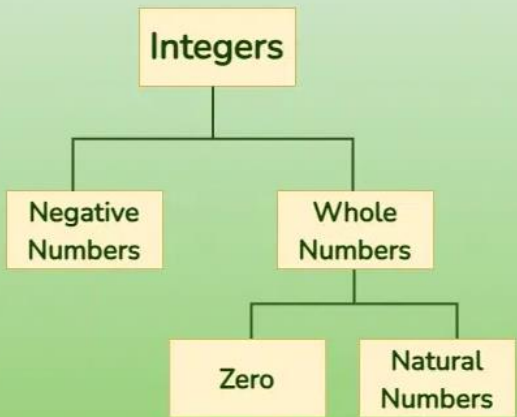
Integers

Definition-

Integers are a set of numbers that include positive whole numbers, their additive inverses (negative whole numbers), and zero.

Examples-

Examples of integers are:
-10, 1, 54, 0, -900, 85, etc.



```
graph TD;
    A[Integers] --> B[Negative Numbers];
    A --> C[Whole Numbers];
    C --> D[Zero];
    C --> E[Natural Numbers];
```

Properties of Addition and Subtraction of Integers

Closure under Addition and subtraction

For every integer a and b , $a+b$ and $a-b$ are integers.

Commutativity Property for addition

for every integer a and b , $a+b=b+a$

Associativity Property for addition

for every integer a, b and c , $(a+b)+c=a+(b+c)$.

Additive Identity

For every integer a , $a+0=0+a=a$ here **0** is Additive Identity, since adding 0 to a number leaves it unchanged.

Example : For an integer 2, $2+0 = 0+2 = 2$.

Additive inverse

For every integer a , $a+(-a)=0$ Here, $-a$ is additive inverse of a and a is the additive inverse of $-a$.

Example : For an integer 2, (-2) is additive inverse and for (-2) , additive inverse is 2.

[Since $+2 - 2 = 0$]

Properties of Multiplication of Integers

Closure under Multiplication

For every integer a and b , $a \times b = \text{Integer}$

Commutative Property of Multiplication

For every integer a and b , $a \times b = b \times a$

Multiplication by Zero

For every integer a , $a \times 0 = 0 \times a = 0$

Multiplicative Identity

For every integer a , $a \times 1 = 1 \times a = a$. Here 1 is the multiplicative identity for integers.

Associative property of Multiplication

For every integer a , b and c , $(a \times b) \times c = a \times (b \times c)$

Distributive Property of Integers

Under addition and multiplication, integers show the distributive property.

i.e., For every integer a , b and c , $a \times (b + c) = a \times b + a \times c$

These properties make calculations easier.

Division of Integers

Division of Integers

When a **positive integer** is divided by a **positive integer**, the quotient obtained is a **positive integer**.

Example: $(+6) \div (+3) = +2$

When a **negative integer** is divided by a **negative integer**, the quotient obtained is a **positive integer**.

Example: $(-6) \div (-3) = +2$

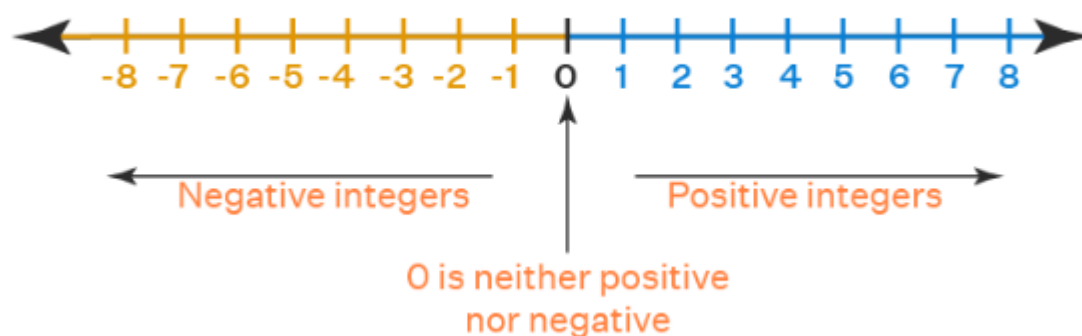
When a **positive integer** is divided by a **negative integer** or **negative integer** is divided by a **positive integer**, the quotient obtained is a **negative integer**.

Example: $(-6) \div (+3) = -2$ and Example: $(+6) \div (-3) = -2$.

PROPERTIES OF INTEGERS				
Property	Operations			
Name	Addition	Subtraction	Multiplication	Division
Closure	$a + b \in \mathbb{Z}$	$a - b \in \mathbb{Z}$	$a \times b \in \mathbb{Z}$	$a \div b \notin \mathbb{Z}$
Commutative	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Associative	$(a + b) + c = a + (b + c)$	$(a - b) - c \neq a - (b - c)$	$(a \times b) \times c = a \times (b \times c)$	$(a \div b) \div c \neq a \div (b \div c)$
Identity	$a + 0 = a$ $0 + a = a$	Not applicable	$a \times 1 = a$ $1 \times a = a$	Not applicable
Distributive	$a \times (b + c) = ab + ac$	$a \times (b - c) = ab - ac$	Not applicable	Not applicable
where $a, b, c \in \mathbb{Z}$			* b is a non-zero integer	

Number Line

Representation of integers on a number line



On a number line when we

(i) add a positive integer for a given integer, we move to the right.

Example : When we add +2 to +3, move 2 places from +3 towards right to get +5

(ii) add a negative integer for a given integer, we move to the left.

Example : When we add -2 to +3, move 2 places from +3 towards left to get +1

(iii) subtract a positive integer from a given integer, we move to the left.

Example: When we subtract +2 from -3, move 2 places from -3 towards left to get -5

(iv) subtract a negative integer from a given integer, we move to the right

Example: When we subtract -2 from -3, move 2 places from -3 towards right to get -1

Addition and Subtraction of Integers

The **absolute value** of +7 (a positive integer) is 7

The absolute value of -7 (negative integer) is 7 (its corresponding positive integer)

Addition of two **positive integers** gives a **positive** integer.

Example : $(+3) + (+4) = +7$

Addition of two **negative integers** gives a **negative** integer.

Example : $(-3) + (-4) = -3-4=-7$

When **one positive** and **one negative** integers are **added**, we take their **difference** and place the sign of the **bigger integer**.

Example : $(-7) + (2) = -5$

For **subtraction**, we add the **additive inverse** of the integer that is being subtracted, to the other integer.

Example : $56 - (-73) = 56 + 73 = 129$